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DISCUSSION PAPER

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DISCUSSION PAPER

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ABSTRACT

Firms' investment decisions involve analyzing prices, products, technologies, productive capacity and the availability of credit. These and other factors were greatly impacted by the 2009 post-crisis economic environment in Brazil. We measure the after crisis impacts of subsidized credit on the optimal level of investment of Brazilian firms from the perspective of the Tobin's q. We combined the Tobin's q framework with the estimation of a panel data stochastic frontier model to establish what optimal levels of investment the subsidized firms should have had. In general the after crisis average-q was very low and it appeared to differ substantially across subsidized and non-subsidized firms. The result indicates a relative disequilibrium between the value of the company and its assets in the post-crisis Brazilian environment. Firms with access to subsidized credits from Brazilian Development Bank (BNDES) did not have higher optimal investment levels, indicating that the crowding out effect can be happening.

1 INTRODUCTION

In the aftermath of the international crisis that hit the Brazilian economy in 2009, the government decided to increase the supply of subsidized credits for firms (Bonomo, Brito and Martins, 2015). This was part of the countercyclical policy that aimed to revive the economy in the short run by increasing the public spending, promoting taxes reductions and by giving direct credits to firms. For that, the government took resources in the market, continuously increasing its budget deficits in the subsequent years, in order to set up the resources that it judged necessary to increase its share in the economy (Almeida, Oliveira and Schneider, 2014). Even though a countercyclical policy should be temporary by definition, in Brazil it lasted until 2014 (Mendes, 2015). During this period the government used the Brazilian Development Bank (BNDES) to give investment allowances to an increasing number of firms (Pinheiro, 2015).

The Brazilian countercyclical policy was an expensive one, and it made the money expensive for everyone else (Almeida, 2015). Together with the post-crisis climate, it influenced the overall availability of credit, the value of savings, the price of goods and the entrepreneurs' expectations over investment projects. In the years following the crisis, the government tried to trim some of these side effects by giving even more subsidized credits, but without making the expected effects (Mendes, 2015). The crisis still had severe effects in the financial sector and reinforced the awareness by the financial institutions (Bonomo, Brito and Martins, 2015). The contagion effect was inevitable and worsened the willingness of financial private institutions to take risk on lending (Almeida, 2015). The increase in state intervention resulted in underperformance of Brazil's economy since 2011 and the government debt reached 63% of the gross domestic product (GDP) in 2014. The economic growth has slowed down and the domestic demand has been affected by decreasing in business sector confidence (Mendes, 2015).

From the neoclassical point of view, the increase in government spending will lead consumers to believe that today's budget deficit will have to be financed by future taxes. In an economy at full employment, this will lead consumers to spend more in present moment and save less for the future (Bernheim, 1989). Interest rates must then rise to bring capital markets into balance. Because of the neoclassical savings-investment identity, this would decrease the investment. The government spending budget will increase the level of consumption, but with side effects over investment in the long run, since the interest rates will also have to increase to balance the decrease in savings. This will make private investments less profitable, and private investments will decrease. Thus, budget deficit triggers a sort of crowding out effect over private investment. Therefore, from the neoclassical point of view, an expansionary fiscal policy has questionable capacity to leverage firm's investments from the beginning (Mendes, 2015).

From the microeconomic point of view the subsidized credit supply would be justified by the existence of markets failures, such as asymmetric information (Antunes, Cavalcanti and Villamil, 2015). Those failures create credit restrictions, pre-empting profitable investments, especially for small and medium firms that cannot get funded in the open market. However, if we assume that large enterprises operate in close to perfect markets (Fazzari, Hubbard and Petersen, 1988), the firms comparison between investments profitability prospects and the cost of replacing the capital would alone lay down the investment decisions. The firm would be indifferent between using their own internal resources and get funded in the open market, with no need from the government to intervene in the credit market supply (Hall and Jorgenson, 1967).

The Tobin's q theory of investment states that the ratio between the value of the company and the replacing price of capital would be the only relevant explanatory variable in an investment equation (Tobin, 1969; Abel, 1981; Modigliani and Miller, 1958). Thus, in a dynamic maximization problem, where the firm takes decisions in order to maximize the future payments to shareholders, will lead to a solution where the Tobin's q alone determines the optimal investment.[1] And if we operate in close to market conditions, we expected this to be also true in the stochastic frontier model. Under this framework we analyzed the effects of post-crisis subsidized credit over the optimal level of investment.[2] We use a sample of enterprises that enables us to compare the evolution of the post-crisis Tobin's q for firms with and without access to the subsidized credits from BNDES. The post-crisis average-q is less than unity for most of the firms in our sample and there appear to be significantly credits restriction effects over investment in the post-crisis Brazilian economy.

After this introduction, in section 2 we provide a discrete version of Tobin's q investment model. In subsection 2.1 we describe the characteristics of our database and some details about the variable definitions. In subsection 2.2 we describe the rereading

1. See section 2 for a detailed explanation about the relationship between Tobin's q and investment.
2. See appendix for a statistical exposure of the stochastic frontier model.

of the stochastic frontier model made by Bhaumik, Das and Kumbhakar (2012) in order to accommodate the optimal investment estimation. In section 3 we provide the parameters estimates for the panel data stochastic frontier models, and in section 4 we state our main conclusions. We also provide in appendix the statistical foundations of the panel data stochastic frontier model.

2 TOBIN'S *Q* AND INVESTMENT ALLOWANCES

There are disagreements in literature over how deeply governments should be involved in credit market supply (Lazzarini *et al.*, 2011). According to the traditional industrial policy framework, the activities of a development bank help to reduce capital constraints and to spur industrial development, as they provide credit to private projects that were not to be undertaken if the subsidized funding were not available (Antunes, Cavalcanti and Villamil, 2015). From the microeconomic point of view, the Tobin's q model puts forward that the firms base their investment decision on future expected earnings. In a frictionless economy the firm's investments should depend only on Tobin's q and market demand (Abel, 1981) and the supply of subsidized credit would not affect the investment (Hayashi, 1982). But since there are no truly frictionless markets (Fazzari, Hubbard and Petersen, 1988), there will always exist installation costs, asymmetric information and credit restraints affecting firm's investment decisions (Modigliani and Miller, 1958). This would justify the existence of subsidized credit supply aiming the increase of the private investment by providing lower tariffs and less financial requirements to enterprises (Buttari, 1995).

In order to show where does the subsidized credit fit in the Tobin's q model, we use the discrete-time version of Abel (1981) and Hayashi (1982) given by professor Christopher Carroll (Carroll, 2014).[3] Let us assume that the firm takes decisions on investments in order to maximize its future payments to shareholders. Define these payments as the present discounted value of after-tax revenues after subtracting the costs of investment (1):

$$e_t(k_t) = \max_{\{i_t\}_{t=0}^{\infty}} E[\textstyle\sum_{n=0}^{+\infty} \beta^n(\pi_{t+n} - \xi_{t+n})] \quad (1)$$

3. We are grateful to professor Carroll for the broad covering of the investment theory in his lectures notes, available on-line at: <http://www.econ2.jhu.edu/people/ccarroll/public/lecturenotes/Investment/>.

Where k_t is the firm's capital stock at time t, i_t is the firm's investments in time t, β is the discount factor, π_t is the after taxes revenue, and ξ_t is the total after-tax spending on investment. Let us introduce a tax rate on corporate finance τ that is applied to revenues and to the depreciation rate δ. We can write the after tax revenue as $\pi_t = (1-\tau)f(k_t)$, where $f(k_t)$ is the gross output excluding investments and other adjustments costs, and π_t is the revenue after taxes. In perfect capital markets $e_t(k_t)$ will be the stock market value of the profit-maximizing firm.

The intertemporal restriction is the next period capital k_{t+1} equal the depreciated current capital plus current investment:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{2}$$

For simplicity, let the shareholders be the suppliers of both physical and financial capital. Then k_t gives the number of shares of stock outstanding in the firm. We can rationalize this by supposing that every time the firm makes an investment it issues the necessary number of shares at a price equal to the marginal valuation of the firm's capital stock. Although the model was designed for open capital firms, those ideas were later applied to private equity firms, by assuming that the private equity entrepreneurs are very aware of their market value and periodically evaluate the pros and cons of going public. So we assume that the entrepreneur in private equity companies also takes actions as to maximize their market value.

Considering the intertemporal equation, the Bellman equation for the firm's present value is:

$$e_t(k_t) = \max_{i_t} \; \pi_t - \xi_t + \beta E_t[\max_{\{i_t\}_{t=1}^{\infty}} \textstyle\sum_{n=0}^{+\infty} \beta^n(\pi_{t+1+n} - \xi_{t+1+n})] \tag{3}$$
$$= \max_{i_t} \; \pi_t - \xi_t + \beta E_t[e_{t+1}((1-\delta)k_t + i_t)] \tag{4}$$

Carroll (2014) introduces an smooth and convex investment adjustment function $j_t = j(i_t, k_t)$. Let ζ be the parameter for investment tax credit, $\tilde{\zeta} = 1 - \zeta$ is the cost of investment after investment tax credit, P_t is the price of one unit of investment, and $\mathcal{P}_t = (\tilde{\zeta} P_t)$ is the effective after-tax price of capital. This is where we point out the effects of subsidized credit in the intertemporal maximization. The subsidized credit will lower the effective after-tax price of capital. But the company has to compare the unit price of a subsidized capital with the expected profits from that capital. Still, the firm would decide not to invest under an unstable economic environment.

The total after taxes investment is $\xi_t = (i_t + j_t)\mathcal{P}_{t+1}\beta$, where:

$$i_t = k_{t+1} - (1-\delta)k_t = k_{t+1} - (1-\delta)k_t.$$

Substituting this back in equation (3), we have:

$$\begin{aligned} e_t(k_t) &= \max_{k_{t+1}}\{\pi_t - [i_t + j_t]\mathcal{P}_{t+1}\beta + \beta E_t[e_{t+1}(k_{t+1})]\} \\ e_t(k_t) &= \max_{k_{t+1}}\{\pi_t - [\underbrace{k_{t+1} - \tilde{\delta}k_t}_{i_t} + \underbrace{j_t(k_{t+1} - \tilde{\delta}k_t; k_t)}_{j_t}]\mathcal{P}_{t+1}\beta + \beta E_t[e_{t+1}(k_{t+1})]\} \end{aligned} \tag{5}$$

Since the government gives credit at lower ζ, the price of a new capital unit $\mathcal{P}_t$ will be lower. Let us define the derivative of adjustment costs with respect to the level of investment as: $\frac{\partial j_t(i_t,k_t)}{\partial i_t} = j^i(i_t, k_t)$. Then the first order condition for optimization with respect to capital (k_t) and investment can be written as:

$$[1 + j_t^{i_t}]\mathcal{P}_{t+1}\beta = \beta E_t[e_{t+1}^k(k_{t+1})] \tag{6}$$

Carroll (2014) interprets (6) by stating that the present discounted value of the marginal investment, after taxes and adjustments costs ($[1 + j_t^{i_t}]\mathcal{P}_{t+1}$), "matches" the marginal expected value of the new capital: $\beta E_t[e_{t+1}(k_{t+1})]$.

Using $\xi = \tau f(k_t)$ and applying the envelope theorem to equation (5), we have:

$$\begin{aligned} &\frac{\partial e_t(k_t)}{\partial k_t} = (1-\tau)\frac{\partial f(k_t)}{\partial k_t} - \frac{\partial j_t(k_t)}{\partial k_t}\mathcal{P}_{t+1}\beta + (1-\delta)\beta E_t[\frac{\partial e_{t+1}(k_{t+1})}{\partial k_t}] \\ &= (1-\tau)f^{k_t}(k_t) - j_t^{k_t}(k_t)\mathcal{P}_{t+1}\beta + (1-\delta)\beta E_t[e_{t+1}^{k_t}(k_{t+1})] \\ &e_t^{k_t}(k_t) = (1-\tau)f^{k_t}(k_t) - [(1 + j_t^{i_t})(1-\delta) - j_t^k]\mathcal{P}_{t+1}\beta \end{aligned}$$

Writing the same equation in the next period $t + 1$ and then expressing (6) as an Euler equation:

$$\begin{aligned} &e_{t+1}^{k_{t+1}}(k_{t+1}) = (1-\tau)f^{k_{t+1}}(k_{t+1}) - [(1-\tau) + (1-\tau)j_t^{i_t} - j_t^k]\mathcal{P}_{t+1}\beta \\ &[1 + j_t^{i_t}]\mathcal{P}_{t+1} = E_t[\tilde{\tau}f^{k_{t+1}}(k_{t+1}) + (\tilde{\delta} + \tilde{\delta}\, j_{t+1}^{k_{t+1}})\mathcal{P}_{t+2}\beta] \\ &= E_t[\tilde{\tau}f^{k_{t+1}}(k_{t+1}) + (\tilde{\delta} + j_{t+1}^{k_{t+1}} + \delta j_{t+1}^{k_{t+1}})\mathcal{P}_{t+2}\beta] \end{aligned} \tag{7}$$

Let us introduce to the maximization problem a "net-investment" function $\tilde{\imath}_t$:

$$\tilde{\imath}_t = \frac{i_t}{k_t} - \delta$$

This is the amount of investment-to-capital diversion from the depreciation rate (δ) that is necessary in order to keep the level of capital constant. Let us take the derivative to the capital and investment:

$$\frac{\partial \tilde{\imath}_t}{\partial i_t} = \frac{1}{k_t}$$

$$\frac{\partial \tilde{\imath}_t}{\partial k_t} = -\frac{\tilde{\imath}_t - \delta}{k_t}$$

where, $i_t = (\tilde{\imath}_t + \delta)k_t$.

The investment-to-capital ratio will be used later as dependent variable in the stochastic frontiers models. Let us define a convex quadratic adjustment function:

$$j_t(i_t, k_t) = \frac{k_t}{2}[\frac{i_t - \delta k_t}{k_t}]^2 \omega = \frac{k_t}{2}\tilde{\imath}_t \omega$$

$$\frac{\partial j_t}{\partial i_t} = (i_t - \delta k_t)\omega$$

$$\frac{\partial j_t}{\partial k_t} = -(\frac{\tilde{\imath}_t{}^2}{2} - \tilde{\imath}_t \delta)\omega$$

And the Euler equation for investment equation (7) will be:

$$(1 + j_t^{i_t})\mathcal{P}_{t+1} = E_t[(1-\tau)f^k(k_{t+1}) + ((1-\delta) + j_{t+1}^i + \frac{\omega}{2}\tilde{\imath}_{t+1}^2)\mathcal{P}_{t+2}\beta]$$

To properly interpret the above equation, let us assume two additional simplifications (Carroll, 2014): *i)* there is no cost of adjustment ($j^i = j^k = 0, \omega = 0$); and *ii)* the price of capital is constant over time ($\mathcal{P}_{t+1} = \mathcal{P}_t = 1$). Let us also use the fact that $1 + r + \delta = 1/\beta(1-\delta)$ to write:

$$\mathcal{P} = E_t[(1-\tau)f^k(k_{t+1}) + \mathcal{P}_{t+2}\beta(1-\delta)]$$

Under no adjustments costs, the unit cost of capital plus the sum of the opportunity cost lost in interest and depreciation must match the after-tax earnings from that capital. The existence of a subsidized credit will affect investments decisions because there will be a lower tax credit for investment (ζ) to compare with the after-tax price of capital ($\mathcal{P}$) (Hall and Jorgenson, 1967).

Let us define $\lambda_t = e_t^k$ as the marginal value to the firm of ownership of one more unit of capital in t. Using the envelope theorem we can write:

$$\lambda_t = (1-\tau)f^k(k_t) + j_t^k \mathcal{P}_t \beta + \beta(1-\delta)E_t[\lambda_{t+1}] \quad (8)$$
$$= (1-\tau)f^k(k_t) + j_t^k \mathcal{P}_t \beta + (1-\delta-r)\{\lambda_t + E_t[\Delta\lambda_{t+1}]\} \quad (9)$$

Using the approximation in equation (8), we have:

$$(r+\delta)\lambda_t = (1-\tau)f^k(k_t) + j_t^k \mathcal{P}_t \beta + E_t[\Delta\lambda_{t+1}] \quad (10)$$

Rearranging the terms, we have:

$$E_t[\Delta\lambda_{t+1}] = (r\lambda_t) - [(1-\tau)f^k(k_t) + j_t^k \mathcal{P}_t \beta - \delta\lambda_t] \quad (11)$$

Equation (11) can be seen as an arbitrage equation for the company's share price in perfect markets. We can compare the flows of income that would be obtained from putting the value of an extra unit of capital in the bank $(r\lambda_t)$ with the flow of having another unit of capital inside the firm $((1-\tau)f^k(k_t) + j_t^k \mathcal{P}_t \beta - \delta\lambda_t)$. The extra revenues ($(1-\tau)f^k(k_t))$ plus the effect of the extra capital on costs of adjustment $(j_t^k \mathcal{P}_t \beta)$ and the term $(\delta\lambda_t)$ will reflect the cost to the firm from the extra depreciation of the new capital.

1) When $E_t[\Delta\lambda_{t+1}] = 0$ the marginal value of capital sunk inside the firm is equal to the opportunity cost of employing that capital in the bank and the firm should be neither growing nor shrinking.
2) When $E_t[\Delta\lambda_{t+1}] > 0$ then $\Rightarrow (r\lambda_t) < ([1-\tau]f^k(k_t) + j_t^k \mathcal{P}_t \beta - \delta\lambda_t)$ then an extra unit of capital is more valuable inside the firm than outside it. The firm should make positive net investments.
3) When $E_t[\Delta\lambda_{t+1}] < 0$ the firm will have incentives to disinvest.
4) Under the framework presented here the availability of subsidized credit will lower the price of a new unit of capital $(\mathcal{P}_t)$ and ameliorate the arbitrage equation in the direction of a positive investment decision.

According to equation (12) the same decision rules can be formulated in terms of the marginal-q. The value of an additional unit of capital inside the firm divided by the after-tax purchase price of an additional unit of capital:

$$q_t = \frac{\lambda_t}{\mathcal{P}_t} \quad (12)$$

Using that is the investment first order condition (6) to write:

$$[1 + j_t^{i}]\mathcal{P}_{t+1}\beta = \beta\lambda_{t+1} \tag{13}$$

$$\frac{\lambda_{t+1}}{\mathcal{P}_{t+1}} = 1 + j_t^{i_t} \tag{14}$$

$$1 + \frac{\partial j_t(i_t, k_t)}{\partial i_t} = q_{t+1} \tag{15}$$

Equation (15) defines an implicit investment equation expressed as function of the marginal-q:

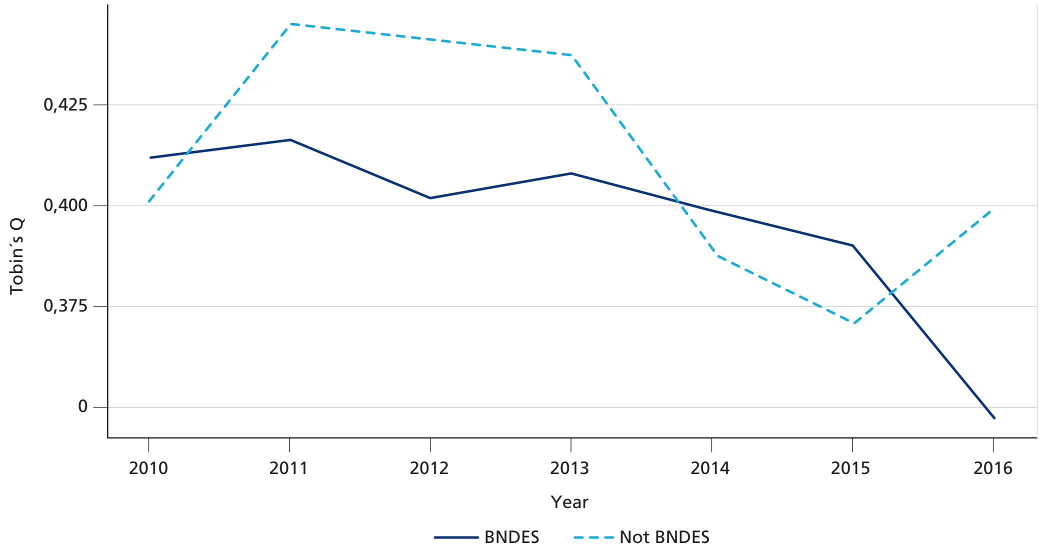
$$\tilde{\imath}_t(q_{t+1}) = \frac{q_{t+1} - 1}{\omega} \tag{16}$$

$$i_t = (\tilde{\imath}_t q_{t+1} + \delta)k_t \tag{17}$$

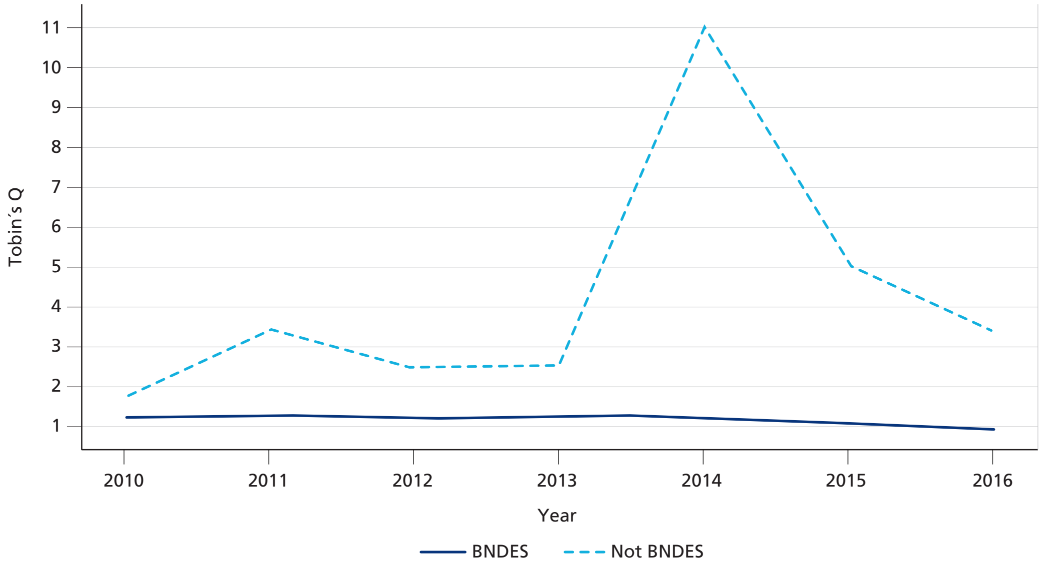
When $q_{t+1} = 1$ investment takes place at a rate equal to the depreciation rate. The investment is monotonically increasing in q_{t+1} and the intensity of the relationship between investment and q_{t+1} will depend on the derivative of the adjustment costs function.

Tobin's q model shows how optimizing firms make investments decisions taking into account the future paths of adjustment costs, the marginal product of a new capital and other features of the market environment. In this framework the subsidized credit from BNDES would influence investments decisions by modifying the effective after-tax price of capital. Besides this justification, the effectiveness of the subsidized credits can be challenged when it is given to large firms or when there is not any follow-up to make sure that those subsidies were used for their stated purposes (Lazzarini *et al.*, 2011).

FIGURE 1
Average Tobin's *q* for subsidized and non-subsidized firms (2010-2016)
1A – Equity over total assets



1B – Equity over fixed assets



Figures 1A and 1B show the evolution of the post-crisis average Tobin's *q* using two alternative measurements for Tobin's *q*.[4] The average-*q* calculated with total assets is lower than the unity for the entire period. The average-*q* calculated with fixed assets is higher than the unit for non-subsidized firms and exhibited a bigger variability.

Following the argument of Ali, Mahmud and Lima (2016), it can be said that we are in a good momentum to invest in Brazilian firms. Since in equilibrium Tobin's *q* tends to 1 in the long run, the value of Brazilian firms is expected to appreciate in the future.

The framework presented in this section has left some unanswered research questions. Since the investment is supposed to happen when *q* exceeded unity, why exactly firms with such a low future expectations of profit in the after-crisis would apply to receive the subsidies? What optimal levels of investment the subsidized firms should have had in post-crisis? Why the firms in the open market are applying for subsidized credit (table 1)?

The literature has already pointed out that the development bank's lending can be a source of credit misallocation in the market, either because they may fail to reach the companies with the finest projects or because of political influence in the credits assignment (Lazzarini *et al.*, 2011). The literature on neopatrimonialism has also mapped how the existence of development banks can bias the credit allocation in the market, contributing to maintain the status quo of large established companies with political power (Altenburg, 2011). Large old companies would have easy access to subsisted credit as they can engage in crony deals with politicians (Kumar, 2014). This can prevent incumbent firms to grow and gain market share, and allow low performance old enterprises with chronic insolvency to survive (Soest, Bechle and Korte, 2011).

2.1 Webscraping database of Brazilian enterprises

In this paper we use a panel data of 500 large Brazilian enterprises, from 2009 to 2016. The database contains a rich set of financial information and was obtained by webscraping techniques using *scrapy-Python*. The final sample contains public traded and closed firms, private and state-owned firms, subsidized by BNDES and non-subsidized by BNDES, national and foreign controlled firms. The dataset covers many sectors (table 1).

4. In the econometric models presented in section 2.2 we only use Tobin's *q* calculated using total assets in the denominator, since this version appeared to be more consistent between groups and over time.

Our response variable in stochastic frontier model is the investment over capital. The investment was computed in the following steps: *i*) compute the firm's total liabilities as the total assets minus the shareholder's equity; *ii*) calculate the sum of current assets and long-term assets as the liquidity value times the total liabilities; *iii*) compute the fixed assets (K_{it}); and *iv*) calculate the investment as: $I_{it} = K_{it} - K_{it-1}$.

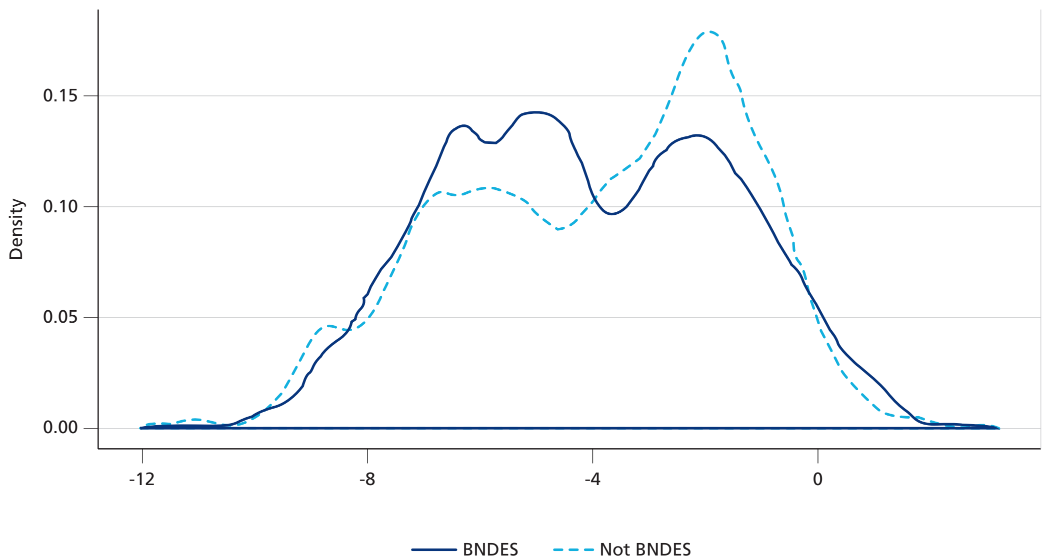
There are different ways to compute the Tobin's q, for instance, there is the average-q (Ali, Mahmud and Lima, 2016), the marginal-q (Matos, 2010) and the fundamental-q (Lorenzoni and Walentin, 2007). Santos *et al.* (2011) provide an overview of different methods for calculating Tobin's q. We chose the average-q, mainly because of the limitations regarding the available variables.

Another important binary variable that will be included in the models is the BNDES. This takes one if the firm had access to BNDES subsidized credits and zero otherwise. We also test the effects from the ratio between the total amount of credit received from BNDES and the capital stock. The rest of the variables included in the stochastic model are listed in table 1.

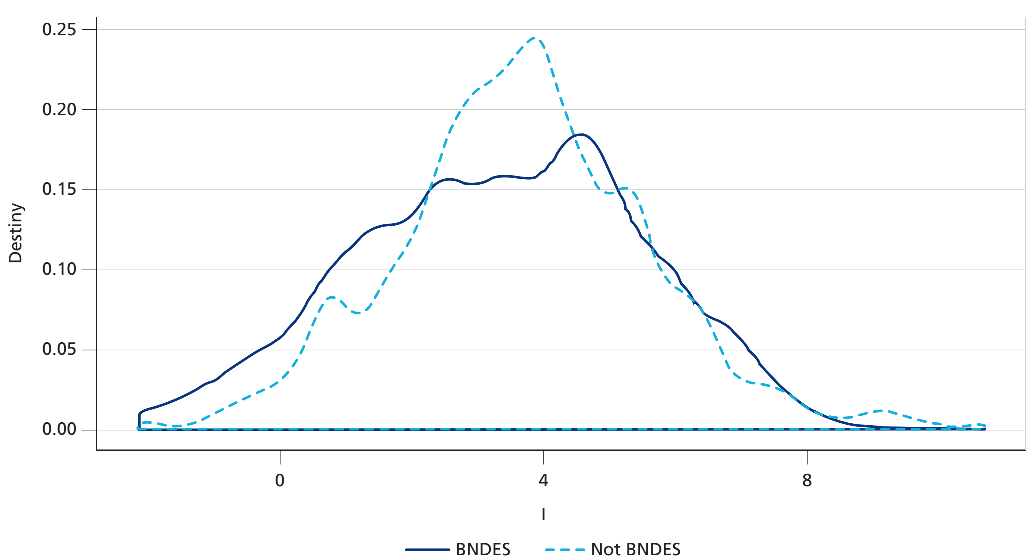
TABLE 1
Summary statistics for Brazilian firms (2009-2016)

	BNDES=1			BNDES=0		
	Mean	Median	Standard deviation	Mean	Median	Standard deviation
Investment/capital	0.243	0.035	1.239	0.168	0.014	0.465
Tobin's q	0.404	0.407	0.176	0.383	0.394	0.350
BNDES over capital	0.895	1.012	1.442	-	-	-
Sale over capital	0.895	1.012	1.442	1.163	1.139	1.765
Cash flow	0.917	0.226	9.108	63.35	0.261	129.3
(log)Assets	6.971	6.872	1.292	6.776	6.696	1.253
(log)Capital	2473.3	370.8	12113	282.9	282.9	5212.2
(log)Equity	1822.1	330.6	9053.3	944.3	288.6	2964.2
Dividends payments (%)	0.467	0.234	0.159	0.265	0.226	1.242
Leverage (%)	0.273	0.245	0.159	0.265	0.230	0.199
Labor	71.22	44.22	44.22	71.22	28.24	143.4
Firms open capital (%)	0.18		0.384	0.073		0.260
Number of firms	843			1416		

FIGURE 2
Distribution of investment/capital: subsidized and non-subsidized firms
2A – Log(Investment/Capital)



2B – Log(Investment)



There is a meaningful variability in investment levels among subsidized and unsubsidized firms and also inside those groups, as it became evident looking at the distance between the mean and median. In general, the subsidized firms (*BNDES=1*) have higher average-*q*, investment levels, equity and leverage. In our sample there is a substantial percentage of publicly traded companies being subsidized by BNDES. We can question why the government gives subsidies to firms that can capture those resources in the open market. Looking at the total number of employees and total assets as proxies for size, the subsidized and unsubsidized firms have the same size in our sample.

2.2 Optimal investment from stochastic frontier model

In section 2 we presented the theory that links the Tobin's *q* with firms' investment decisions. Now we measure the impacts of subsidized credits, together with Tobin's *q* and other variables over the level of investment for Brazilian enterprises. We calculate average Tobin's *q* according to the approach taken by Blundell *et al.* (1992) and Ali, Mahmud and Lima (2016). The remaining explanatory variables are based on the work of Bhaumik, Das and Kumbhakar (2012) and Fazzari, Hubbard and Petersen (1988).

If a firm is not financially constrained, then its investment decisions should be entirely captured by the Tobin's *q* (Lindenberg and Ross, 1981) and maybe its past sales. Otherwise, if a firm's investments are significantly related to cash flow, then the firm may be operating under a credit constraint (Ciaian and Swinnen, 2009; Blundell *et al.*, 1992). The statistical significance of the cash flow in investment models is confirmed by a large number of empirical articles (Mukherjee, 2015). With that in mind, the cash flow and other variables are included in the part of stochastic frontier model that compounds the efficiency equation. Other variables that could capture market failures environment characteristics are size, leverage and dividends payments (Fazzari, Hubbard and Petersen, 1988).

We start with the stochastic frontier panel data model of Bhaumik, Das and Kumbhakar (2012):

$$Y_{it} = \exp(x_{it}\beta + \vartheta_{it} + \mu_{it}) \tag{18}$$

Where $Y_{it} = I_{it}/K_{it-1}$ denotes the investment in time t divided by capital stock (K_{it-1}) in *t-1*; x_{it} is a vector of known functions of inputs of investment and other explanatory variables in the main equation; β is a vector of parameters to be estimated and μ_{it} is truncated non-negative technical inefficiency.

The technical inefficiency in equation (18) is a random effect with mean $z_{it}\delta$ and variance σ^2, where z_{it} is a vector of explanatory variables associated with technical inefficiency over time; and δ is a vector of unknown parameters to be estimated. Equation (18) contains a random i.i.d. error term independent ϑ_{it} that is independent of μ_{it}. In turn, the technical inefficiency effect μ_{it} in equation (18) could be specified as:

$$\mu_{it} = z_{it}\delta + \omega_{it} \tag{19}$$

Where ω_{it} is the truncation of the normal distribution with zero mean and variance σ^2.

Fazzari, Hubbard and Petersen (1988) also point out that other variables such as leverage (L_{it}), dividends payments (D_{it}), past sales (S_{it}) and cash flow (CF_{it}), would capture the effects of market frictions. In perfect markets, the estimated parameters (δ) in the inefficiency equation (19) would be null. Since we are testing for the credit constrains and other market frictions, we choose to include in x_{it} : past sales over capital (S_{it}/K_{it-1}), past subsidized credit loans over capital ($loan_{it}/K_{it-1}$), dividends payments (D_{it}), leverage ($\beta_3 D_{it}$) and year dummy. In z_{it} we include the cash flow over capital (CF_{it}/K_{it-1}), total assets ($assets_{it}$), a dummy variable for access to BNDES loans ($BNDES_{it}$) and a dummy variable for access to open capital market ($OPEN_{it}$).

In particular the cash flow is included in the stochastic part of the frontier model that compounds the efficiency equation. Agency conflicts models (Childs, Mauer and Ott, 2005) justify the inclusion of dividends payments (D_{it}) in the investment equation (18), since the underinvestment or overinvestment disequilibrium created by stockholder-bondholder conflicts can be highlighted under market failures.

Assuming logarithmic relation between average-q and investment, then the model (18) becomes:

$$\ln\left[\frac{I_{it}}{K_{it-1}}\right] = \beta_0 + \beta_1 \ln(Q_{it}) + \beta_2\left(\frac{S_{it-1}}{K_{it-1}}\right) + \beta_3 D_{it} + \beta_3 L_{it-1} + \vartheta_{it} + \mu_{it} \tag{20}$$

$$\mu_{it} = z_{it}\delta + \omega_{it} \tag{21}$$

The explanatory variables in the inefficiency equations are known as the *z*-variables in equation (22), intended to measure credit restrictions characteristics.

$$\mu_{it} = \delta_0 + \delta_1(\frac{CF_{it}}{K_{it-1}}) + \delta_2 \ln(A_{it}) + \delta_3 \text{BNDES}_{it} + \delta_4 \text{OPEN} + \omega_{it} \tag{22}$$

The rationality of stochastic frontier model is to attribute high efficiencies to someone that exhibits the highest levels of outputs with the lowest possible inputs (x_{it}). Since our output is investments, we can treat the amount of subsidized credit as an input, putting it in the main equation. Thus, the non-subsidized firms will appear with zero input of subsidized credit.

The term $[I_{it}/K_{it-1}]^{sf}$ represents the optimal unobserved investment-to-capital ratio, while ($\ln[I_{it}/K_{it-1}]$) represents the observed investment-to-capital ratio. The difference between the optimal and observed investment-to-capital ratio is the non-negative inefficiency term μ_{it}.

Let us write the observed investment-to-capital ratio as:

$$\ln[\frac{I_{it}}{K_{it-1}}] = \ln[\frac{I_{it}}{K_{it-1}}]^{sf} - \mu_{it} \tag{23}$$

By assuming that the past capital K_{it-1} is optimal (Bhaumik, Das and Kumbhakar, 2012), we can simplify equation (23) and interpret $(I_{it}/I_{it}^{sf}) = \exp(-\mu_{it})$ as the investment efficiency. The term μ_{it} measures the shortfall of investment from its desired level. The higher the value of μ_{it} the greater the impact of constraints on investment. The statistical proprieties of the panel data stochastic frontier model (Battese and Coelli, 1995) are presented in more details in the appendix.

3 RESULTS

Table 2 shows the different econometric models structures tested in this section. The aim is to check how much the inclusion or omission of some variables, as well as the use of different stochastic models, would affect the estimated parameters and the optimal efficiency levels.

TABLE 2
Model Specification

Model	Type	Variables
1 and 2	Linear fixed effect model	$y_{it} = \alpha + \boldsymbol{x_{it}\beta} + \lambda_i + \zeta_t - \xi_{it}$
3 and 4	Stochastic frontier: Battese and Coelli (1995)	$y_{it} = f(x_{it}, \beta)\exp(v_{it} - \mu_{it})$ $\mu_{it} = \exp(-\eta[t - T])\ \mu_i$ $\mu_i \sim N(\mu, \sigma^2)$ $v_{it} \sim N(0, \sigma_v^2)$
5, 6, 7 and 8	Stochastic frontier: Greene (2005a) True fixed and random effect	$y_{it} = (\alpha + \omega_i) + f(x_{it}, \beta) + v_{it} - \mu_{it}$ $\mu_{it} \sim N^+(0, \exp(\omega_{it0} + \mathbf{z'}_{\mathbf{\mu it}}\boldsymbol{\omega_{it}}))$ $v_{it} \sim N(0, \sigma_v^2)$ $\omega_i \sim N(0, \sigma_\omega^2)$

Table 3 shows that the Tobin's q is positively related to the investment as expected. In the fixed effect model, lacking the truncated structure of the stochastic frontier model, the effect of Tobin's q seems to be exacerbated. An increase of 1% on Tobin's q seems to lead to an increase of 0.5% over the investment. For instance, percentage of dividends payments affects negatively the optimal investment, whereas leverage affects positively the optimal investment.

The stochastic frontier model of Battese and Coelli allows some variables to be included in the inefficiency equation to capture the effects of market frictions over investment. We notice that all the variables in the inefficiency equation showed statistical significance. The size of the companies, represented by total assets, showed negative sign, indicating that size seems to alleviate the credit constrains. The negative sign of the cashflow variable and the dummy variable for BNDES credit access indicates that those variables diminish the effects of market frictions (table 3). In the main equation the increase of 1% in the Tobin's q leads to an increase of 0.32% in the optimal investment.

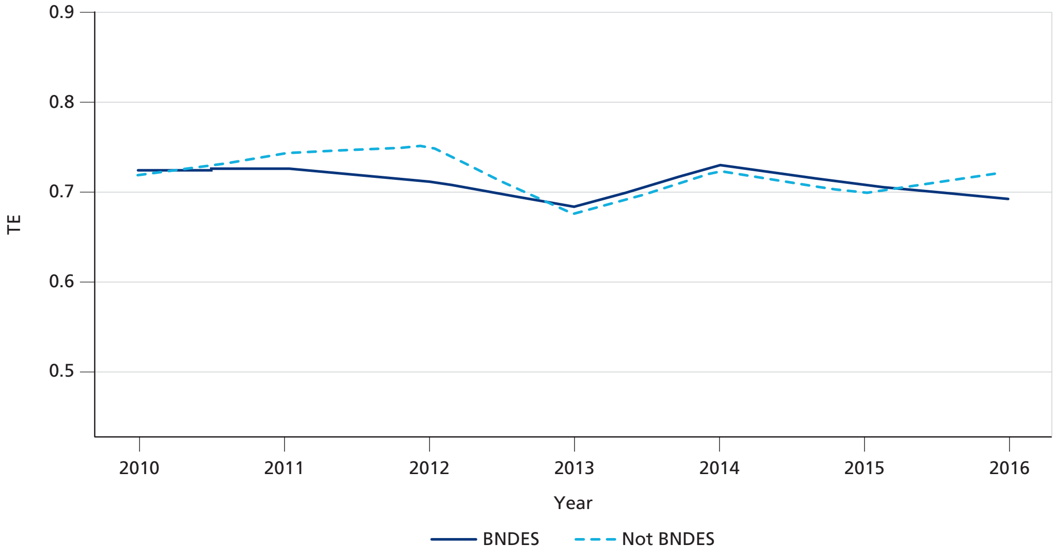
We did not notice from figures 3 and 4 the presence of any substantial gap in the efficiency levels between firms subsidized by BNDES and firms non-subsidized by BNDES. At the same time, open capital companies appear to have inferior levels of investment efficiency in comparison to closed capital firms (figure 3B – appendix). Fazzari, Hubbard and Petersen (1988) point out that investment's announcements may have a negative effect over the firm value under certain circumstances. We would expect that, since foreign firms are more exposed to external volatile climate they may choose to undergo investments in local plants. But from our analysis, it appears that the presence in open market makes enterprises more sensitive to a crisis environment. Foreign and national firms did not have any gap in the levels of efficiency (figure 4).

TABLE 3
Linear panel data and stochastic frontier models (2009-2016)

Explanatory variables	Fixed effects		Battese and Coelli (1995)	
	(1)	(2)	(3)	(4)
β_0	-5.93***	-5.85***	-3.99***	-4.21***
	(0.713)	(0.715)	(0.176)	(0.150)
$\ln(Q_{it})$	0.50*	0.51*	0.26**	0.32*
	(0.278)	(0.278)	(0.099)	(0.097)
$\ln(S_{it-1}/K_{it-1})$	0.19***	0.15***	0.54***	0.75***
	(0.03)	(0.034)	(0.049)	(0.006)
$\ln(loan_{it-1}/K_{it-1})$	-	0.005	-	0.029***
	-	(0.004)	-	(0.006)
D_{it}	-0.03	-0.03	-0.07**	-0.07***
	(0.07)	(0.07)	(0.023)	(0.023)
L_{it}	0.83***	0.84***	0.12	0.25
	(0.306)	(0.306)	(0.402)	(0.393)
δ_0	-	-	0.35	0.39
			-	-
$\ln(CF_{it}/K_{it-1})$	-0.029	-0.030	-0.26***	-0.28***
	(0.058)	(0.058)	(0.064)	(0.055)
$\ln(assets)$	0.85***	0.84***	0.01	-0.06*
	(0.107)	(0.108)	(0.052)	(0.030)
$BNDES_{it}$	0.196**	0.135	-0.56***	-0.14***
	(0.104)	(0.115)	(0.108)	(0.095)
OPEN	-0.79***	-0.77***	0.13	0.15*
	(0.197)	(0.197)	(0.199)	(0.101)
σ_μ	1.10	1.09	0.0061	0.0038
σ_v	-	-	2.09	2.09
Log-likelihood	-2320	-2319	-3999	-3992
AIC	4670	4670	8021	8009
BIC	4755	4760	8087	8081

Obs.: *p<0.1; **p<0.05; ***p<0.01.

FIGURE 3
Stochastic frontier efficiency gap (2010-2016)
3A – Subsidized and non-subsidized firms



3B – Publicly traded and private

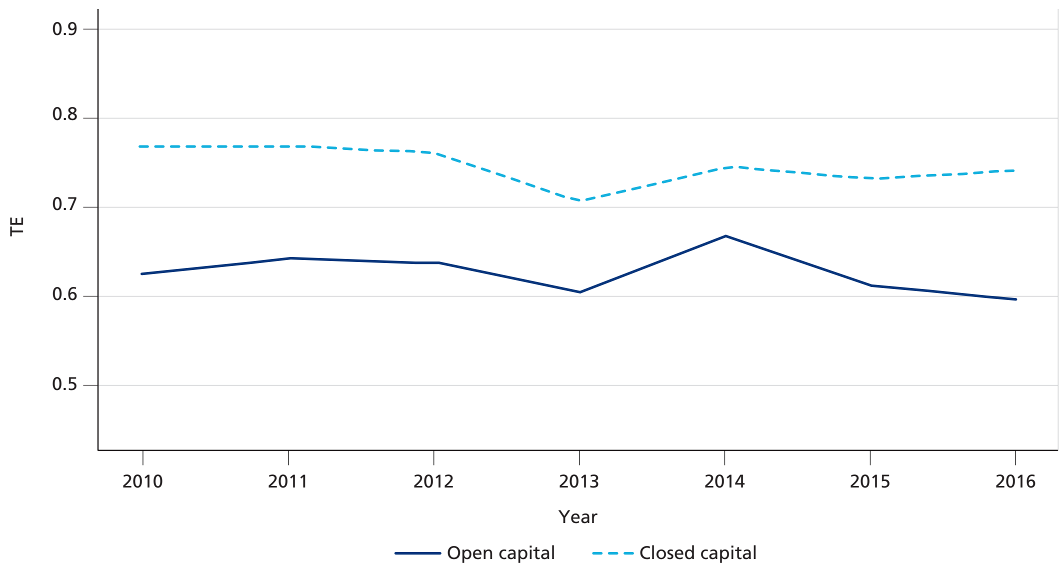
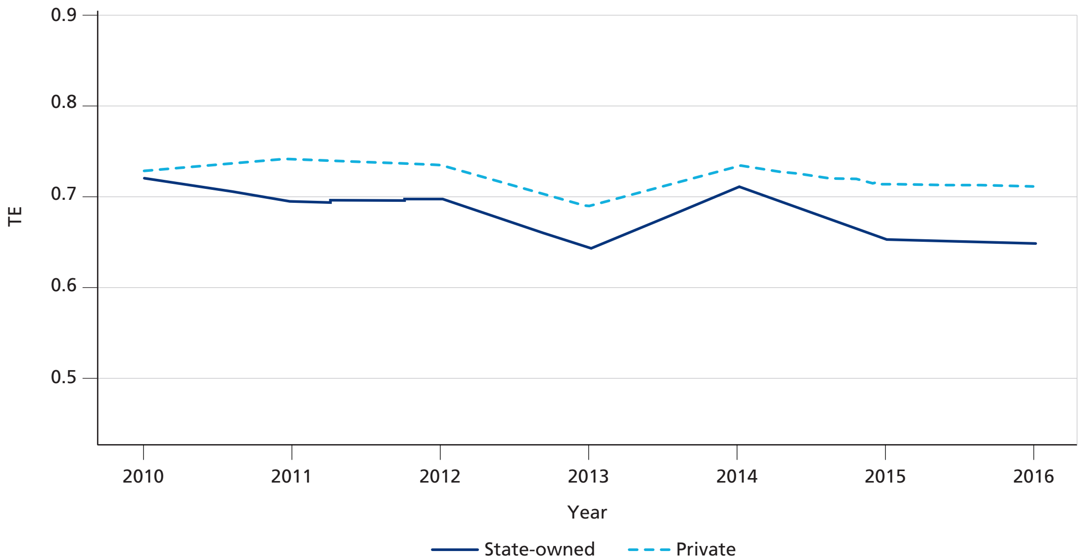
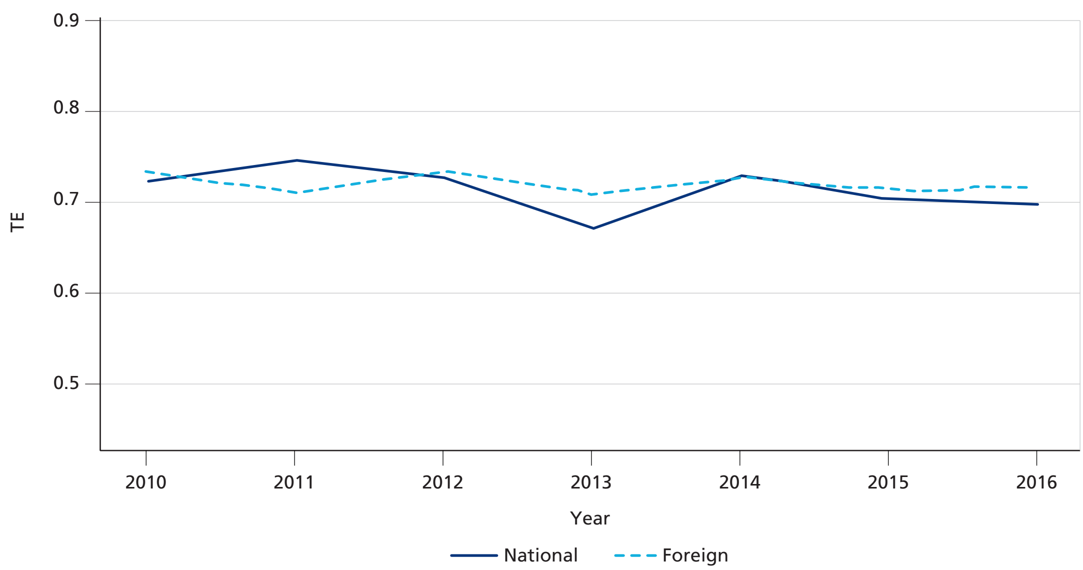


FIGURE 4
Stochastic frontier efficiency gap (2010-2016)
4A – State owned and private firms



4B – National and foreign firms



In general, the firms that received subsidized credit from BNDES did not appear to have higher optimal levels of investment. This is a strong indication of the crowding out hypothesis. However, we should conduct further investigations to confirm this.

We found a significant percentage of open capital firms that were subsidized by BNDES (table 1). Those firms should be able to capture resources in the open market if they found a friendly economic environment. One can also point out that those subsidized loans should be given to medium small enterprises, and we find a significant percentage of large enterprises that has accessed those subsidies.

Contrary to the stylized facts in literature, the marginal-*z* effect (table 4) for total assets confirms that the smaller the firm's size is the smaller is the investment's inefficiency. The positive marginal effect in the upper part of the cash flow distributions could be an indication of crowding out effect, since those firms should have had lower levels of investment constrains.

TABLE 4
Marginal effects of *z* variables in Battese and Coelli's model

Statistics	BNDES	Cash flow	Ln(labour)
Minimum		-0.022	-0.009
Percentile 25%		0.130	0.053
Median	Firms subdized by BNDES	0.151	0.061
Percentile 75%		0.194	0.079
Maximum		0.255	0.104
Minimum		-0.125	-0.051
Percentile 25%		0.079	0.032
Median	Firms non-subsidized by BNDES	0.129	0.052
Percentile 75%		0.194	0.079
Maximum		0.254	0.103

For a matter of consistency, we also choose to show the parameter estimates from GREENE'S (2005B) true fixed and true random panel data stochastic frontiers models (table 5). We notice that the parameter estimated for Tobin's *q* is always positive and has statistical significance in all the models presented in table 5. This is in conformity with the results from Battese and Coelli (1995) models presented in table 3. Together with Tobin's *q*, leverage was positive and significant in all the models. The dummy variable for BNDES seems to increase the variability of the stochastic inefficiency term, while the dummy variable for open market seems to decrease the variability of the stochastic inefficiency term.

TABLE 5
Greene stochastic frontier models for panel data (2009-2016)

		GREENE (2005A) True fixed effects		GREENE (2005A) True randon effects	
		(5)	(6)	(7)	(8)
Frontier	$\ln(Q_{it})$	0.337**	0.349***	0.333**	0.342**
		(0.131)	(0.131)	(0.144)	(0.143)
	$\ln(S_{it-1}/K_{it-1})$	0.236***	0.228***	0.240***	0.236***
		(0.014)	(0.014)	(0.015)	(0.0148)
	$\ln(loan_{it-1}/K_{it-1})$		0.007**		0.004***
			(0.002)		(0.002)
	D_{it}	-0.102***	-0.099***	-0.065*	-0.0645***
		(0.040)	(0.040)	(0.038)	(0.0382)
	L_{it}	0.834***	0.842***	0.761***	0.769***
		(0.156)	(0.156)	(0.150)	(0.150)
σ_μ	$\ln(CF_{it}/K_{it-1})$	3.158***	3.189***	3.111***	3.123***
		(0.137)	(0.138)	(0.132)	(0.133)
	$\ln(assets_{it})$	1.034***	1.032***	1.065***	1.059***
		(0.057)	(0.058)	(0.058)	(0.058)
	$BNDES_{it}$	3.062***	2.186***	3.050***	3.119***
		(0.251)	(0.255)	(0.262)	(0.264)
	$OPEN_{it}$	-3.350***	-3.343***	-3.019***	-2.995***
		(0.250)	(0.252)	(0.205)	(0.206)
	δ_0	-11.53***	-11.66***	-11.70***	-11.74***
		(0.560)	(0.563)	(0.564)	(0.565)
	$BNDES_{it}$			-0.511***	-0.508***
σ_v^2				(0.098)	(0.098)
	$OPEN_{it}$			-0.946***	-0.919***
				(0.095)	(0.0956)
	ω_0	-2.050***	-2.050***	-1.625***	-1.636***
		(0.042)	(0.042)	(0.062)	(0.0617)
Log-like		-1747	-1464		
AIC		4496	3935		
BIC		7308	6758		

Obs.: 1. Standard errors in parentheses.
2. *p<0.1; **p<0.05; ***p<0.01.

Despite the desired consistency of the results along different models specifications, we must point out that the true random effect stochastic frontier model lacks consistency when the dataset is composed by many observations in a small period of time (Belotti *et al*., 2012; Kumbhakar, Lien and Hardeker, 2014).

The positive sign and statistical significance of past sales ($\ln(S_{it-1}/K_{it-1})$) seems to be consistent across the diferent specitications of true fixed and random effects models, confirming the existence of significant credit constrains in the market. In all the true fixed and random effect models, the cash flow ($\ln(CF_{it}/K_{it-1})$) and total assets seem to increase the variability of the stochastic term for the inefficiency in investment.

Overall, the results presented in table 5 confirmed the positive relation between the Tobin's q and investment levels in the post-crisis Brazilian economy. On the other side, the parameter estimated for the amount of loans from BNDES showed a very low estimative, but still showed statistical significance. This shows that the subsidized credit from government does not seem to relate with the optimal investment levels, at least in the same intensity of the Tobin's q, leverage and past sales.

4 CONCLUSIONS

In this paper we report evidences that the Tobin's q explains a meaningful fraction of investment levels of large Brazilian enterprises. In accordance with the literature, we found that Tobin's q has positive sign in all stochastic frontier models. The subsidized credit did not appear to explain the optimal investment levels in the same intensity as the Tobin's q, past sales and cash flow in all the econometric specifications.

We did not find a meaningful gap in the optimal levels of investment between firms subsidized by BNDES and firms non-subsidized by BNDES. For instance, in the true fixed effects stochastic frontier models, the Tobin's q is always positive and significant. However, in the Battese and Coelli models, the Tobin's q lost its significance after we put into the model the amount of subsidized credit resources. We found statistical significance of the variables indented to capture the effects of market frictions, conforming that credit restriction played an important role during the post-crisis period in Brazil.

In the after-crisis period the subsidized credit supply was not able to attenuate the credit restrictions and did not have an effect over the optimal investment frontier. This suggests that development banks should not be used as a tool for prolonged countercyclical polices, as the persistence to intervene in credit market could be a source of even more market frictions.

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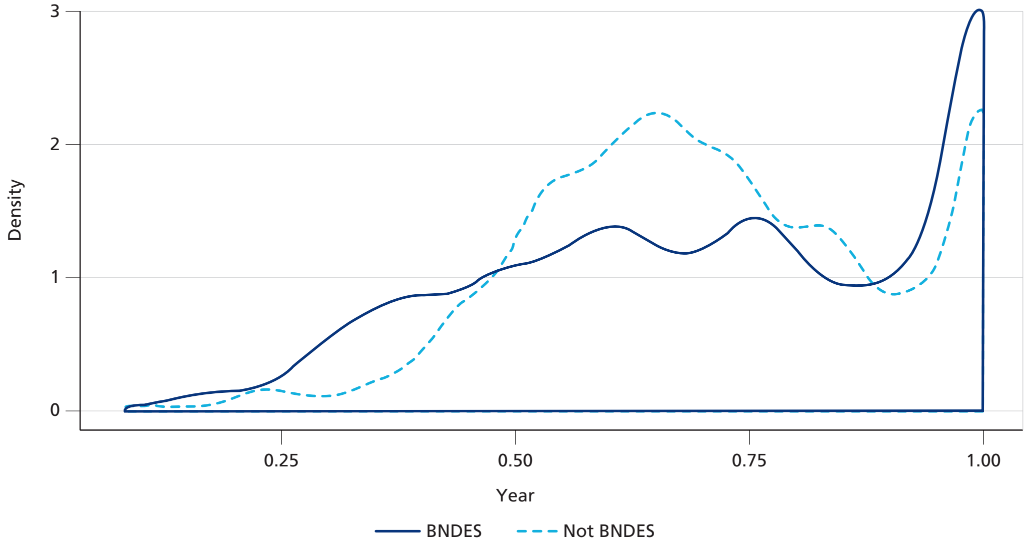
APPENDIX A

INEFFICIENCIES DISTRIBUTIONS

FIGURE A.1
Inefficiencies distributions

A.1A – Subsidized and non-subsidized firms



A.1B – Publicly traded and private equity firms

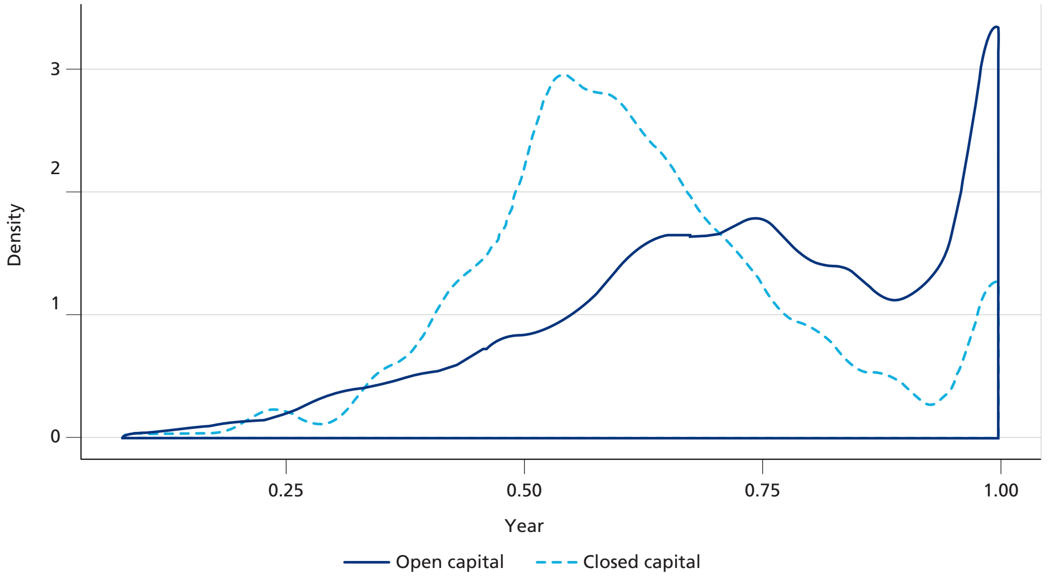
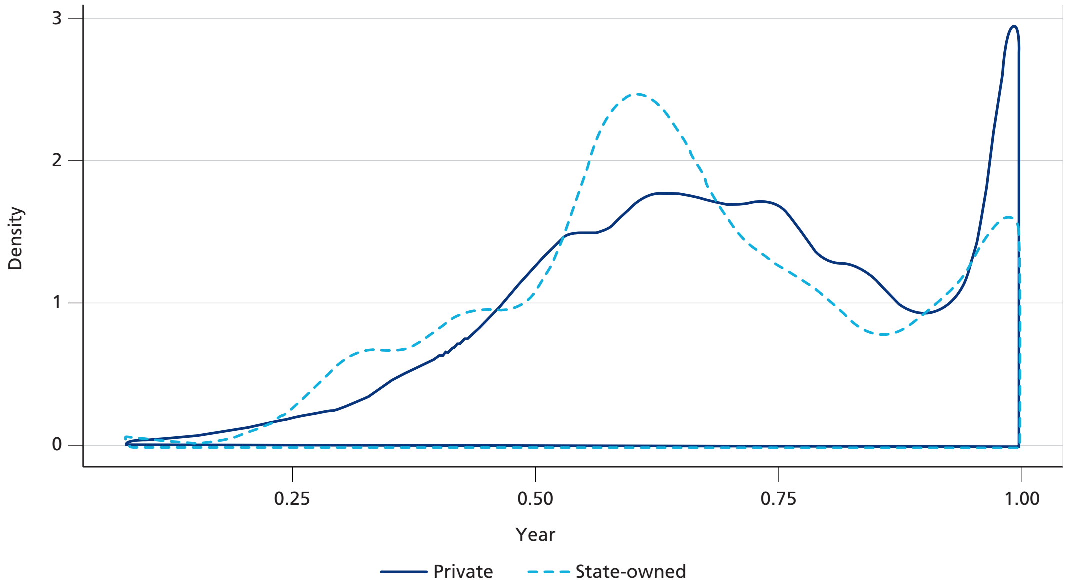


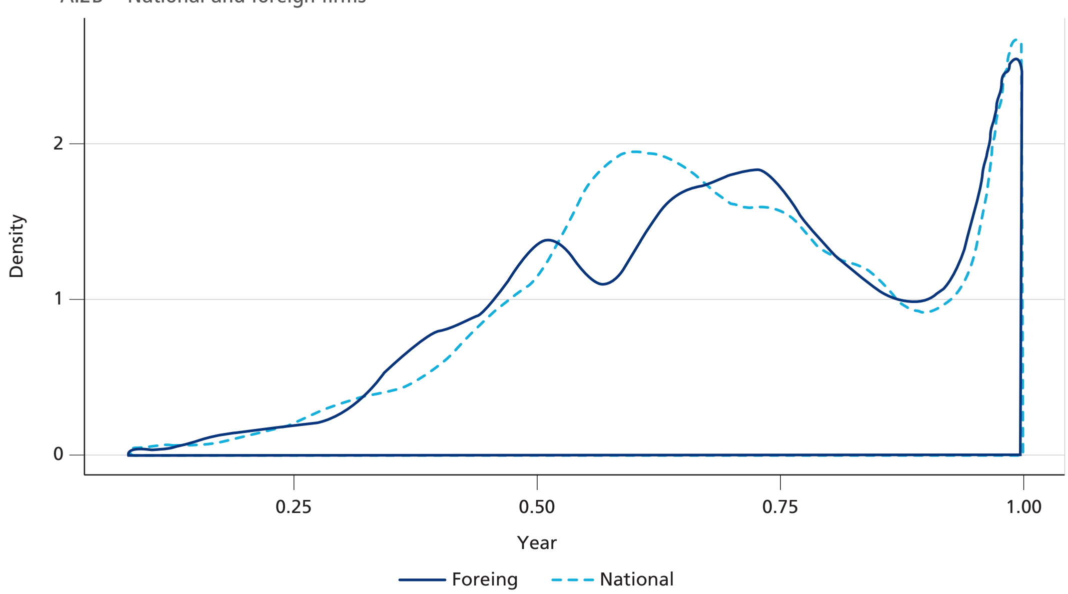
FIGURE A.2

Inefficiencies distributions

A.2A – State owned and private firms



A.2B – National and foreign firms



APPENDIX B

STOCHASTIC FRONTIER MODEL FOR PANEL DATA

Stochastic production functions made important contributions to the estimation of technical efficiency of companies. Battese and Coelli (1992) define the firm's technical efficiency as the ratio of the observed level of the interest-dependent variable (output or cost) divided by the observed level if the firm made efficient use of its resources. The stochastic boundaries considered will account for two components: *i*) the presence of technical inefficiency; and *ii*) a random error term.

Considering a stochastic production frontier with exponential specification over the effects of time varying variables, we incorporate the panel data structure by considering firms in periods. Let us define the equation:

$$Y_{it} = f(X_{it}, \beta)\exp(V_{it} - U_{it})$$
$$U_{it} = \eta_{it} U_i = \exp[-\eta(t - T)]U_i$$

Where Y_{it} is the firm's production; $f(X_{it}, \beta)$ is an unknown function of the parameters β and vector of variables X_{it}. The term V_{it} is i.i.d with normal distribution; $(V_{it} \sim N(0, \sigma_V^2)), U_{it}$) is i.i.d with non-negative truncated normal distribution; $(U_{it} \sim N(\mu, \sigma_U^2))$ is some unknown parameters; and $\kappa(i)$ represents the set of time periods T_i. Let us define a frontier production function as:

$$Y_{it} = X_{it}\beta + V_{it} - \eta_{it} U_i$$

Where $\eta_{it} = e^{-\eta(t-T)}, t \in \kappa(i)$. The non-negative truncated normal distribution for U_i is given by:

$$f_U(u_i) = \frac{[-\frac{1}{2\sigma^2}(u_i - \mu)^2]}{\sqrt{2\pi}\sigma[1 - \phi(-\mu/\sigma)]}, \qquad u_i \geq 0$$

We can represent cumulated normal distribution by $\Phi(\cdot)$, where $\phi = \Phi'$. It can be shown that the mean $(E(U_i))$ and variance $(Var(U_i))$ of this truncated normal is given by:

$$E(U_i) = \mu + \sigma\{\frac{\phi(-\mu/\sigma)}{1 - \Phi(-\mu/\sigma)}\}$$
$$Var(U_i) = \sigma^2\{1 - \frac{\phi(-\mu/\sigma)}{1 - \Phi(-\mu/\sigma)}[\frac{\mu}{\sigma} + \frac{\phi(-\mu/\sigma)}{1 - \Phi(-\mu/\sigma)}]\}$$

Now let V_i be a vector of size $(T_i \times 1)$ representing the random variable V_{it} for each T_i, and let E_i be a vector of size $(T_i \times 1)$ taking values from the join distribution (U_i, V_i):

$$\psi(\eta_i, u_i) = \frac{(e_i+\eta_i u_i)'(e_i+\eta_i u_i)}{\sigma_V^2}$$

$$\varphi(u_i) = \frac{(u_i-\mu)^2}{\sigma^2}$$

$$f_{U,E}(u_i, e_i) = \frac{\exp\left(\frac{-1}{2}(\varphi(u_i)+\psi(\eta_i))\right)}{(2\pi)^{T_i+1}\sigma\sigma_V^{T_i}[1-\Phi(-\mu/\sigma)]}$$

Where e_i is the actual value for the random variable E_i. The density function for E_i is obtained by integrating the distribution $f_{U,E}(u_i, e_i)$ over U_i.

$$\nu(\eta_i, u_i) = e_i{}'e_i/\sigma_V + (\mu/\sigma)^2 - (\mu_i^*/\sigma_i^*)^2$$

$$f_E(e_i) = \frac{1-\Phi\left(\frac{-\mu^*}{\sigma^*}\right)\exp\left(\nu(\eta_i, u_i)\right)}{(2\pi)^{T_i/2}\sigma_V^{T_i-1}(\sigma_V^2+\eta_i{}'\eta_i\sigma^2)^{1/2}[1-\Phi(-\mu/\sigma)]}$$

$$\mu_i^* = \frac{\mu\sigma_V^2-\eta_i e_i\sigma^2}{\sigma_V^2+\eta_i{}'\eta_i\sigma^2}$$

$$\sigma^* = \frac{\sigma^2\sigma_V^2}{\sigma_V^2+\eta_i{}'\eta_i\sigma^2}$$

The density function for the vector Y_i of dimension $T_i \times 1$ is given by the substitution of $y_i - x_i\beta$ by e_i in $f_E(e_i)$, where x_i is a matrix of size $T_i \times k$ and k is the size of the parameter's matrix β. The log-likelihood that we should maximize over the sample distribution $y = (y_1{}', \cdots, y_N{}')$ is given by:

$$\ln[L(\theta^*, y)] = -\frac{1}{2}[\sum_{i=1}^{N} T_i]\ln(2\pi) - \frac{1}{2}[\sum_{i=1}^{N} (T_i - 1)]\ln(\sigma^2) - \frac{1}{2}\sum_{i=1}^{N} \ln(\sigma_V^2 + \eta_i{}'\eta_i\sigma^2)$$
$$-N\ln(1 - \Phi[-\mu/\sigma]) + \sum_{i=1}^{N} \ln(1 - \Phi[-\mu^*/\sigma^*]) - \frac{1}{2}\sum_{i=1}^{N} [(y_i - x_i\beta)'(y_i - x_i\beta)]$$
$$-\frac{1}{2}N\left(\frac{\mu}{\sigma}\right) - \frac{1}{2}\left(\frac{\mu^*}{\sigma^*}\right)$$

In Battese and Coelli (1992) model the parameters estimated by the maximization of the above likelihood is such that the non-negative firm effects (U_{it}) only decrease, increase or remain constant over time. The exponential specification for the firms behavior over time is specified according to: $U_{it} = \exp[-\eta(t - T)]U_i$. As this can be such rigid parametrization for the technical efficiencies, greater flexibility can be obtained with a double parameter specification, according to:

$$\eta_{it} = 1 + \eta_1[t - T] + \eta_2[t - T]^2$$

Where η_1 and η_2 are the unknown parameters. This specification allows for the convexity or concavity of firms effects. The time invariant model is a particular case when $\eta_1 = \eta_2 = 0$. When we have an transcendental of Cobb-Douglas production function, then E_{it} is a linear function of the vector β. The technical efficiency in time (t $(TE_t = E[\eta_t U_i])$ is obtained by integrating over the density of U_i:

$$TE_t = \{\frac{1-\Phi[\eta_{it}\sigma-(\mu_i/\sigma_i)]}{1-\Phi[-\mu_i/\sigma_i]}\}\exp(-\eta_t\mu + \frac{1}{2}\eta_t^2\sigma^2).$$

Battese and Coelli (1995) make several extensions to the stochastic frontier methodology for panel data. For example, they allow for non-negative technical inefficiencies to be a specific function of time and firm.

The technical inefficiencies have truncated normal distribution, where the mean and variance are function of observable variables. The firms stochastic frontier evolves according to their own explanatory variables. The extension of Battese and Coelli (1995) can be taken as a more complete and adequate to panel data. Let us assume the following production function for panel data:

$$Y_{it} = \exp(x_{it}\beta + V_{it} - U_{it})$$

Equation $Y_{it} = \exp(x_{it}\beta + V_{it} - U_{it})$ specifies a production frontier in terms of its original explanatory variables. The technical inefficiencies U_{it} can be specified as:

$$U_{it} = nz_{it}\delta + W_{it}$$

Where W_{it} is the truncated normal distribution with zero mean and variance σ^2. The point of truncation $-z_{it}\delta$ is given by $W_{it} \geq -z_{it}\delta$. Those assumptions are consistent with a truncated non-negative normal distribution $(U_{it} \sim N(z_{it}\delta, \sigma^2))$. The likelihood is specified in terms of both parameters of variance: $\sigma_S^2 = \sigma_V^2 + \sigma^2$ and $\gamma = \frac{\sigma^2}{\sigma_S^2}$. The technical inefficiencies are given by:

$$TE_{it} = \exp(-z_{it}\delta - W_i)$$

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